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HIGH RESOLUTION DYNAMICS LIMB SOUNDER

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Subject/title: S/N performance of synchronous sampling as a method of demodulation

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Compares the signal and noise performance of the proposed method of synchronous sampling followed by a digital FIR filter with synchronous demodulation followed by signal integration. Concludes that the ultimate performance with equivalent bandwidths is identical, and that the correct formula for the NEP at the detector in either case (assuming only detection of the fundamental) is

$$\frac{1}{C_f R}\sqrt{\frac{dv_n^2}{df}\Delta f}$$

where C_f is the amplitude of the fundamental in the Fourier series of the chopping waveform, R is the responsivity, dv_n^2/df is the noise density around the chopper frequency and Δf is the noise bandwidth of the preamp, sampling waveform and following filter considered as a composite filter at the chopping frequency. If Δf is essentially governed by the smoothing of the following filter, then it is also equal to twice the noise bandwidth of that filter considered as a filter at zero frequency, for obvious reasons. As usual, the NEN is given by

$$NEN = \frac{NEP}{A\Omega\tau}$$

where $A\Omega$ is the etendue and τ is the optics transmission.

Key words:

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Aim

Calculate signal/noise ratios for signal chains consisting of

1) chopper–detector–preamplifier–synchronous demodulation–low pass filter.
2) chopper–detector–preamplifier–synchronous sampling–digital FIR filter.

Notation

P Input radiant power on detector (chopper fully open − chopper fully closed).

C Chopping waveform $C(t)$ at frequency f_c, with Fourier coefficients C_n:

$$C(t) = \sum_{n=-\infty}^{\infty} C_n \exp i2\pi n f_c t.$$

The fundamental amplitude is given by $C_f = 2|C_1|$. For a square wave, with symmetry about $t = 0$, $C_0^{sq} = \frac{1}{2}$, $C_n^{sq} = \frac{\pm 1}{n\pi}$ for $n =$ odd, with C_1 positive, signs alternating, and $C_{-n} = C_n$. For a trapezoidal waveform with rise time (and fall time) equal to τ, $C_n^{trap} = C_n^{sq} \operatorname{sinc} n\pi f_c \tau$

R Detector responsivity

$G(f)$ Preamplifier gain as a function of frequency

H Output of preamp $H(t)$, with Fourier transform $H(f)$.

D Demodulation waveform $D(t)$ with Fourier coefficients D_n. The only relevant one is unit switching in phase with the $C(t)$, for which the coefficients are $D_0 = 0$, $D_n = 2C_n^{sq}$.

S Sampling waveform $S(t)$ with Fourier transform $S(f)$.

d, F The S waveform consists of a single cycle $d(t)$ with Fourier transform $d(f)$, repeated several times (possibly with varying weights), described by a sequence of delta functions $F(t)$, with Fourier transform $F(f)$. $S(t)$ is $d(t)$ convolved with $F(t)$, so that $S(f) = d(f)F(f)$.

dv_n^2/df Mean square noise density at detector due to background and detector noise. Assumed to be white noise with $1/f$ noise excess at low frequency.

c FIR filter coefficients.

1. Signal/noise calculation for synchronous demodulation.

The detected signal is $PRC(t)$, which is periodic with Fourier coefficients PRC_n. These are amplified by the preamp to give $PRC_nG(nf_c)$. This signal is then multiplied in the time domain, that is convolved in the frequency domain, with the demodulation waveform D. We are interested in the signal around zero frequency, which is

$$\text{Signal} = \sum_{n=-\infty}^{\infty} PRG(nf_c)C_nD_{-n}. \tag{1}$$

There are also ripple components of the signal at f_c, $2f_c$ etc.

Noise power in the input at f is convolved with D to appear in the output at $f-nf_c$, $(n=-\infty\ldots\infty)$. Thus noise power in the output at f derives from input noise power at different frequencies:

$$\left.\frac{dv_n^2}{df}\right|_{out} = \sum_{n=0}^{\infty} \left.\frac{dv_n^2}{df}\right|_{in} |G(nf_c)|^2(|D_n|^2) + |D_{-n}|^2). \tag{2}$$

Note that since $D_0 = 0$, the $1/f$ noise at low frequency does not appear at the output around $f=0$, but at f_c, $3f_c$ etc., which is the reason for using synchronous detection in the first place. (This also implies that the ripple components of the signal at these frequencies are contaminated with $1/f$ noise, and so only the low frequency signal ouput is useful. Thus the demodulation must be followed by a low pass filter, such as integration for a time T.) Thus for an output noise power around zero frequency, the input noise powers in the summation in (2) are all around multiples of the chopper frequency, and are thus assumed to be constant factors in the sum. Two trivial examples as a check on the formalism are as follows.

a) **Square wave choping and wideband preamp.**

Under these conditions we can put $G(nf_c) = G$ and evaluate the sums:

$$\text{Signal} = PRG \sum_{n=-\infty}^{\infty} C_n^{sq} D_{-n} = PRG \sum_{n=\text{odd}} \frac{4}{n^2\pi^2} = \frac{1}{2}PRG \tag{3}$$

which is obviously the correct answer for a rectified square wave. The noise power is given by

$$\left.\frac{dv_n^2}{df}\right|_{out} = G^2 \left.\frac{dv_n^2}{df}\right|_{in} \sum_{n=-\infty}^{\infty} |D_n|^2 = G^2 \left.\frac{dv_n^2}{df}\right|_{in} \tag{4}$$

indicating that the noise power density around zero frequency is the same as the white noise power density at frequencies above the $1/f$ noise in the input, apart from the preamp gain. This is also obvious since the effect of multiplying by ± 1 in the demodulation does not change the rms value of the noise. However it does conceal a subtlety which is illustrated by Figure 1. (The value of f_c is taken to be 500 Hz in this and subsequent figures.) Input noise power in a bandwidth Δf around multiples of f_c appears at different frequencies in the output as shown, and if the input noise is all white, the output in each band has the same value. However, by the usual definition of noise bandwidth, the output band at zero frequency only has half the bandwidth. Ignoring the difference between $8/\pi^2$ and 1 we can simplify by saying that half the noise power in a band Δf around f_c appears in a band of half the width at zero frequency (and therefore with the same density), and half in a band of width Δf at $2f_c$ (with half the noise density, the other half here coming from input noise at $2f_c$). If the following filter has a noise bandwidth Δf_{DC} the NEP is found by setting Signal in (3) equal to rms noise in (4):

$$NEP = \frac{2}{R}\sqrt{\frac{dv_n^2}{df}\Delta f_{DC}}. \tag{5}$$

For example, integration for a time T gives a noise bandwidth $1/2T$, and

$$NEP = \frac{2}{R}\sqrt{\frac{dv_n^2}{df}\frac{1}{2T}}.$$

b) **Square wave chopping followed by detection of fundamental**

For these conditions we assume the preamp gain falls to zero between f_c and higher harmonics: $G(f_c) = G$, $G(nf_c) = 0$ for $n > 1$. There are then no sums to evaluate:

$$\text{Signal} = PRG(C_1^{sq}D_{-1} + C_{-1}^{sq}D_1) = \frac{4}{\pi^2}PRG \tag{6}$$

$$\left.\frac{dv_n^2}{df}\right|_{out} = (|D_1|^2 + |D_{-1}|^2)G^2 \left.\frac{dv_n^2}{df}\right|_{in} = \frac{8}{\pi^2}G^2 \left.\frac{dv_n^2}{df}\right|_{in}. \tag{7}$$

Again this is easy to understand: the fundamental is a sine wave with amplitude $2/\pi$ which gets rectified by D, and the average of the rectified output gives another factor of $2/\pi$. The noise is also smaller, because we are now sensitive only to noise around f_c, but the reduction is small because $(|D_1|^2 + |D_{-1}|^2)$ contains $8/\pi^2$ of the total summation. The NEP with a low pass filter with noise bandwidth Δf_{DC} is correspondingly increased, by a factor $\pi/2\sqrt{2}$:

$$NEP = \frac{\pi}{\sqrt{2}R}\sqrt{\frac{dv_n^2}{df}\Delta f_{DC}}. \tag{8}$$

c) **General results**

Applying the above formalism with confidence we obtain for the NEP in general

$$NEP = \frac{1}{R}\sqrt{\frac{dv_n^2}{df}\Delta f_{DC}}\,\frac{\sqrt{\sum_{-\infty}^{\infty}|G(nf_c)|^2|D_n|^2}}{\sum_{-\infty}^{\infty}G(nf_c)C_nD_{-n}} \tag{9}$$

which for detection of the fundamental only reduces to

$$NEP = \frac{1}{C_fR}\sqrt{\frac{dv_n^2}{df}2\Delta f_{DC}} \tag{10}$$

of which (8) is a special case for square wave chopping ($C_f = 2/\pi$).

2. Signal/noise calculation for synchronous sampling.

We can look at the second half of the signal chain in Section 1 in a different way, by viewing the demodulation by D and low pass filtering as a single operation on the output of the preamp H, which produces an output signal of which we take one sample, so that the mean value of the output is the signal, and the total mean square noise in the output is the signal variance. The operation is described by a sampling function S:

$$\text{Output}(t=0) = \int H(t)S(t)\,dt = \int_{-\infty}^{\infty} H(f)S(-f)\,df.$$

We can view $S(t)$ as the convolution of one cycle of $D(t)$, which we define to be $d(t)$, convolved with a sequence of delta functions of various weights $F(t)$, describing the low pass filtering. (This is a rather approximate representation of the low pass filter except for the special case of integration for an integer number of chopper cycles, when it is exact, with F consisting of N delta functions of equal weight.) $S(f)$ is thus given by the product $F(f)d(f)$. With this way of looking at the signal chain we can find the signal by putting H equal to the signal output of the preamp:

$$\text{Signal} = \sum_{n=-\infty}^{\infty} PRG(nf_c)C_n d(-nf_c)F(-nf_c) \tag{11}$$

and the noise by putting H equal to the amplified noise:

$$\left.\frac{dv_n^2}{df}\right|_{out} = \int_0^{\infty} \left.\frac{dv_n^2}{df}\right|_{in} |G(f)|^2|d(f)|^2|F(f)|^2\,df. \tag{12}$$

Since $F(t)$ consists of a sequence of delta funtions spaced by $1/f_c$, $F(f)$ consists of a sequence of sharply peaked functions around nf_c. Noting also that $d(nf_c) = D_n$, we see that (11) and (12) are essentially equivalent to (1) and (2), with the final averaging over several chopper cycles described by F included in the calculation. Sensitivity to higher harmonics (in either signal or noise) is thus entirely controlled by the combination of preamp gain G and d. If we assume that this product is negligibly small at multiples of f_c above the fundamental, then the only signal is at f_c, and it is sensible to define the noise bandwidth of the composite filter $G\,d\,F$ as

$$|G(f_c)|^2|d(f_c)|^2|F(f_c)|^2\Delta f = \int_0^{\infty} |G(f)|^2|d(f)|^2|F(f)|^2\,df$$

We can then evaluate the NEP by setting Signal equal to rms noise:

$$NEP = \frac{1}{C_f R}\sqrt{\frac{dv_n^2}{df}\Delta f} \tag{13}$$

which is equivalent to (10) because the noise bandwidth of the composite filter at f_c is twice the value of the noise bandwidth of the low pass filter considered in section 1.

This way of looking at the signal chain permits a very simple transition to a sampled system: we simply change the function d describing one chopper cycle of sampling function from a continuous function to delta functions at the time of the samples, and the coefficients of the delta functions in $F(t)$ are just the FIR filter coefficients c. Equation (13) for the NEP still applies to the case where there is no detection of higher harmonics, as do the above remarks concerning the rejection of those harmonics by $G(f)$ and $d(f)$, and noise away from f_c by $F(f)$. With this in mind it is useful to plot the part played by d and F in this process in different schemes, to indicate the performance required of G. Figure 2 compares $d(f)$ for the demodulation and sampling approaches. The upper panel of Figure 2 shows $|d(f)|$ for two different relative phasings between the integration period and the demodulation waveform, realisable by starting and ending the integration at one of the plus to minus transitions (solid curve) and midway between two transitions (dashed curve). The lower panel shows the analagous plot for sampling: the solid curve shows $|d(f)|$ for a sampling cycle consisting of (chopper open – previous chopper closed), and the dashed curve a cycle where the chopper closed value is the mean of the preceding and following samples. These plots clearly indicate the problem of noise at multiples f_c aliasing into the signal band, and also show that the sampling approach is more sensitive to this than the demodulation approach. The reason for the preference that has already been expressed (TC-NCA-18) for the symmetrical sampling function is also apparent — it gives better rejection of noise at low (as opposed to zero) frequency, and around $2f_c$. It is worth noting that these considerations tend towards the definition of G as a bandpass filter centred on f_c, in which case the signal output of the preamp is a sine wave, and there is only one correct choice of time at which to sample it (unless much more complicated, switching filter designs are being considered, which would seem to compromise the simplicity of the analoge electronics brought by the sampling approach). This also implies that timing of the sampling, or phasing relative to the chopper is both important (the NEP formulae given here are for optimum phase) and in principle subject to change if there is change in chopper motion characteritics, or phase shifts in G. Thus some degree of post-launch phase adjustment ought probably to be allowed for.

Finally Figure 3 shows the performance of two very simple FIR filters. They are both 15 tap filters, symmetric in time, with coefficients generated by simple algorithms about which I do not make any claims at all except that they produce numbers with exact binary representations! The upper panel shows the coefficients, and the lower panel $F(f)$. The solid curves are for $c_n = 1 - (n/8)^2$, and the dashed curves $c'_n = c_n^2$. (The $F(f)$ curves have been renormalised by $\sum c$ to make the peaks equal to unity.) These plots illustrate the general behaviour of $F(f)$, periodic with period f_c. Also the FIR filter contributes nothing to rejection of $1/f$ noise, having a pass band centred at zero frequency. The noise bandwidth in each period of $F(f)$ is $f_c \sum c^2/(\sum c)^2$, which is 38 Hz and 45 Hz for the two filters shown in Figure 3. The plots also illustrate the tradeoff between peak sharpness and ringing: the solid curve has a narrower peak, and overall smaller noise bandwidth, but markedly more ringing. The NEN requirement in the IRD I take to be for a notional FIR filter describing integration for a time VIFOV/(Scan Rate), for which Δf_{DC} in (10) is 7.8 Hz, or Δf in (13) is 15.7 Hz.

FIGURE 1

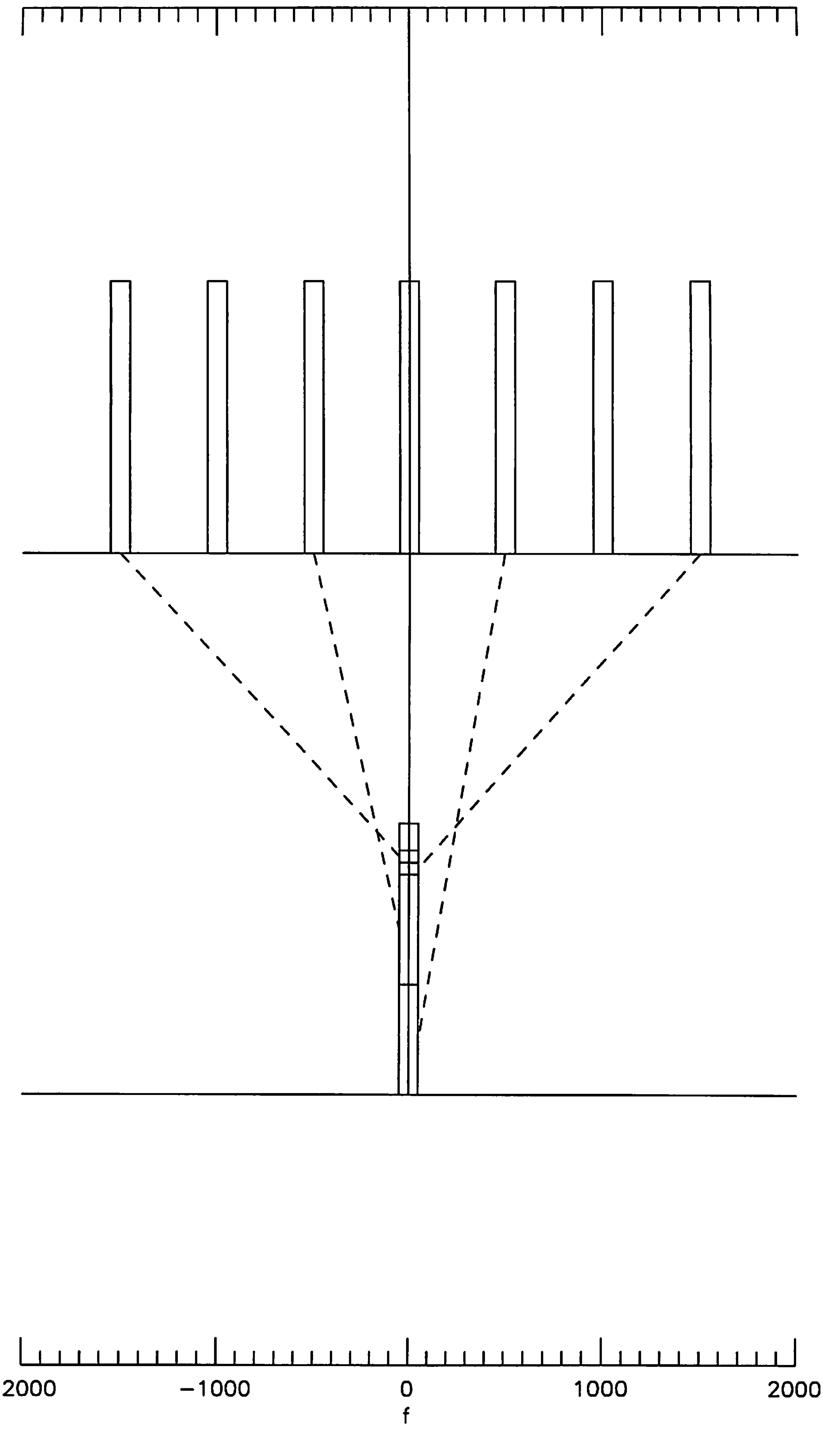


FIGURE 2

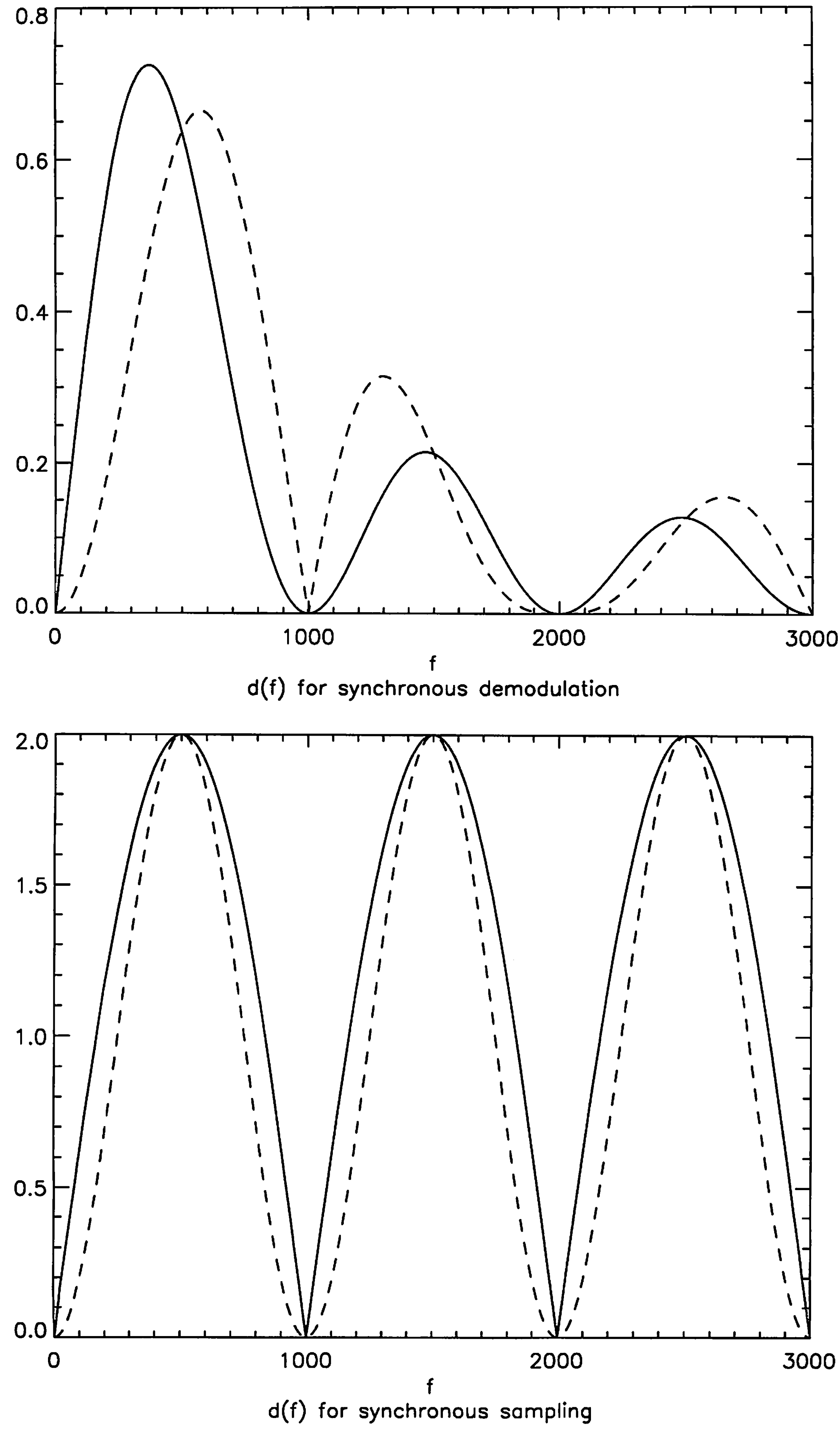
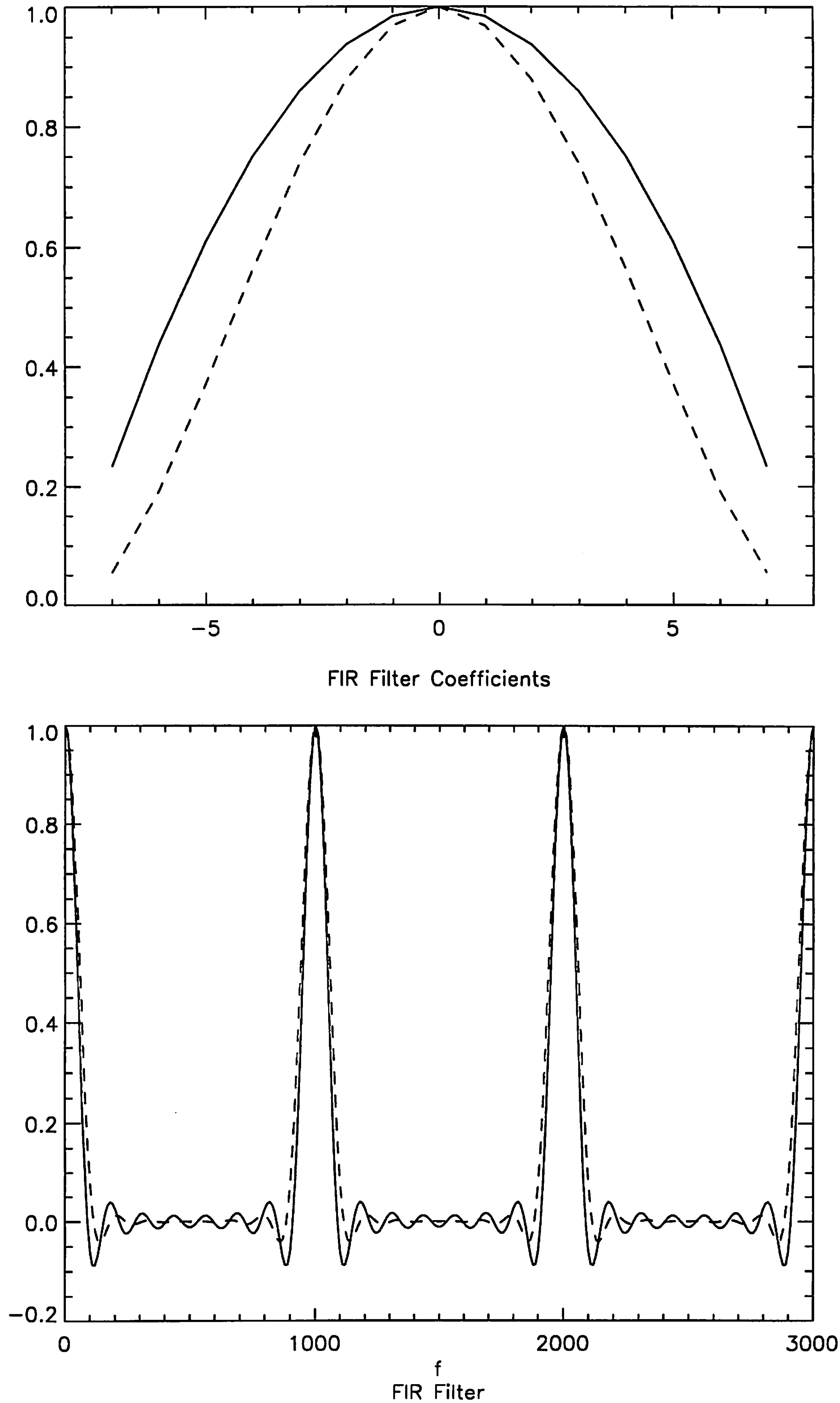

0.8
0.6
0.4
0.2
0.0
0
1000
2000
3000
f
d(f) for synchronous demodulation
2.0
1.5
1.0
0.5
0.0
0
1000
2000
3000
f
d(f) for synchronous sampling

FIGURE 3


1.0
0.8
0.6
0.4
0.2
0.0
−5
0
5
FIR Filter Coefficients
1.0
0.8
0.6
0.4
0.2
0.0
−0.2
0
1000
2000
3000
f
FIR Filter

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Subject: My last word on sqrt(2)

On the basis of Jeanne's helpful account of how the people measuring it define NEP I agree that TC-OXF-52 is in error. The key equation is the general result (9), in which the term in the sum in the denominator should be squred, and the sum square rooted. The special case (10) then looses the factor of 2 in the square root.

CWPP